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METHODICAL FEATURES OF CONSTRUCTING A CLASSICAL MECHANICS COURSE USING THE METHOD OF GEOMETRIC IDEAS

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Abstract: The article discusses the application of the method of geometric ideas in the methodology of teaching classical mechanics. Based on the features of geometric spaces that underlie Newtonian mechanics, Lagrange mechanics, and Hamiltonian mechanics, the relationship of laws, principles, and equations on which these basic approaches to the description of classical dynamical systems are based is shown. The scheme of mutual transitions between different spaces and the corresponding approaches is given.

Keywords: physics teaching methodology, classical mechanics, Euclidean geometry, configuration space, phase space.

As well as Mechanics in the course of general physics, Classical Mechanics is the first section of Theoretical physics course that students begin to study. The foundations laid during its study are fundamentally important when considering all subsequent sections (Electrodynamics and SRT, Quantum Mechanics, Thermodynamics and Statistical Physics).

As you know, the basis of Classical mechanics is formed by three different approaches to the description of dynamical systems: Newtonian mechanics, Lagrange mechanics and Hamilton mechanics. In the traditional approach to teaching this discipline, the study begins with Newtonian mechanics then the Lagrange and Hamilton approaches are sequentially considered. Each of the approaches is based on its geometric space, and this circumstance, from our point of view, is a key point in choosing a specific method for solving physical problems.

Namely, Newtonian mechanics is a theory based on flat Euclidean space; Lagrange mechanics is built on a configuration space characterized by generalized coordinates; Hamilton's mechanics are built on a phase space. Unfortunately, these geometric features, as a rule, either receive very little attention in the educational process, or they are not considered at all. At the same time, most students in practical classes, if there is no specific instruction from the teacher, have difficulties with the choice of mechanics that would allow them to most effectively solve a particular problem. Of course, the equality of these three approaches is a fundamental point, but their existence would not make sense if their differences did not give advantages when considering certain tasks.

We believe that the principles and concepts of geometry can be used to solve the above-mentioned problem of choice. Namely, it is not necessary to attach the space to the corresponding mechanics, but, on the contrary, first consider the geometric side of the process and choose the space that will best characterize the object, and then write down the corresponding equation of mechanics. Geometry is a very obvious science, and therefore it is very likely to say that with this approach, it will be much easier for students to analyze the occurring phenomena and find solutions [1, 2]. This approach is seen as particularly relevant in light of current trends in the geometrization of physics in general (see, for example, [3, 4]).

Let us consider in more detail the formation of a mechanistic picture of the world using the method of geometric ideas.

At the beginning of the study of the course, it is necessary to introduce students to the basic geometric concepts, definitions, various spaces and methods of transition between them. For example, one of the simplest spaces is the space in which the Euclidean geometry is valid. With this space, you can associate a Cartesian coordinate system. To determine the position of a moving object, 3 coordinates (x, y, z) are used in it. If it is still necessary to specify the orientation of the object, then it is necessary to introduce already 6 coordinates ($x, y, z, \varphi, \theta, \psi$).

The space in which Euclidean geometry is valid is modeled by a geometric set of points, it is continuous, homogeneous, isotropic, simply connected and has 3 dimensions, and the distance between any two points in it is determined by the usual formula:

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 = \Delta l^2 .$$

This geometry is very convenient to use if simple problems are considered, in which you can accurately predict which coordinates will describe the movement of the object, and how the forces will act in the system under consideration. If the answer to these questions is obvious and not difficult, then it is necessary to use Newtonian mechanics, which suggested that the fundamental physical quantities $\vec{r}$, t and m (space, time and mass, respectively) are absolute, i.e. do not depend on the speed of physical objects:

$$\frac{d\vec{r}}{dt} = \vec{\upsilon}, \qquad \frac{d\vec{\upsilon}}{dt} = \vec{a},$$

$$\int d\vec{r} = \vec{r} = \int \vec{\upsilon} dt + const, \qquad \int d\vec{\upsilon} = \vec{\upsilon} = \int \vec{a} dt + const.$$

The conserved quantities, such as momentum, angular momentum, and energy are determined by the equations:

$$\vec{P} = m\vec{\upsilon}, \qquad \vec{M} = [\vec{r}, \vec{P}], \qquad E = \frac{m\upsilon^2}{2} + mgh.$$

which result from combinations $\vec{r}$, t and m through differentiation and integration. The basic equation of Newtonian dynamics is

$$m\vec{a} = \vec{F}.$$

If the answer to these questions causes difficulties for a number of reasons (e.g. it is inconvenient to use Cartesian coordinates, it is impossible to specify all the forces acting in the system, especially the reaction forces etc.), then any parameters are introduced to describe the object that will allow it to be characterized. It can be areas, angles, lengths, etc. These selected parameters (or one parameter) are called generalized coordinates describing the configuration of the system, and the corresponding space will be called configuration. When choosing this space, the Lagrange mechanics is used to solve the problem, the equation of motion of which in differential form has the form:

$$\frac{d}{dt}\left(\frac{\partial L}{d\dot{q}_j}\right) - \frac{\partial L}{dq_j} = Q_j,$$

where $L = L(q, \dot{q}, t)$ is the Lagrange function, which is the difference of kinetic and potential energies:

$$L = \frac{p^2}{2m} - U(q).$$

Quite often, it is necessary to consider problems in which it is not just the movement of the system, but the changes in its state over time have to be defined. Then it is necessary to add special coordinates to the “pure” geometric generalized coordinates, which will describe the state of the moving object itself. These parameters are momentums. This space (state space) will be called phase space, and the line that the moving object will describe therein will be called the phase trajectory. In the phase space, the geometric and physical parameters of the system are combined. And due to this, it is possible to determine not just the position, but the state of the system or point at a certain moment of time. In this case, when solving problems, it is more convenient to use the equations of Hamilton mechanics:

$$\dot{p} = -\frac{\partial H}{\partial q}, \qquad \dot{q} = \frac{\partial H}{\partial p},$$

where H is a time-depended Hamilton function $H = H(q, p, t)$; $H = E_{kinetic} + E_{potentil}$, i.e. it represents the sum of the kinetic and potential energies; p is a generalized momentum, $p = p_j$, $j = 1, 2, ..., s$, where s is number of degrees of freedom; $q = q_j$ are the generalized coordinates.

Thus, it is seen that the most common is the phase space, upon simplification of which (i.e., discarding the physical characteristics), we obtain a configuration space. Euclidean space can be considered as a special case of configuration space.

Also, in the first lessons, it is necessary to immediately show that the Lagrange mechanics is obtained from Newtonian mechanics, and the Hamilton mechanics equations are derived from the Lagrange equations. For the transition between the equations of different mechanics, the principle of least action, the D'Alembert – Lagrange principle, the Maupertuis principle are used. After this, each of the mechanics can be examined in more details.

The diagram of mutual transitions between different spaces and the corresponding mechanics is presented in the figure 1 below.

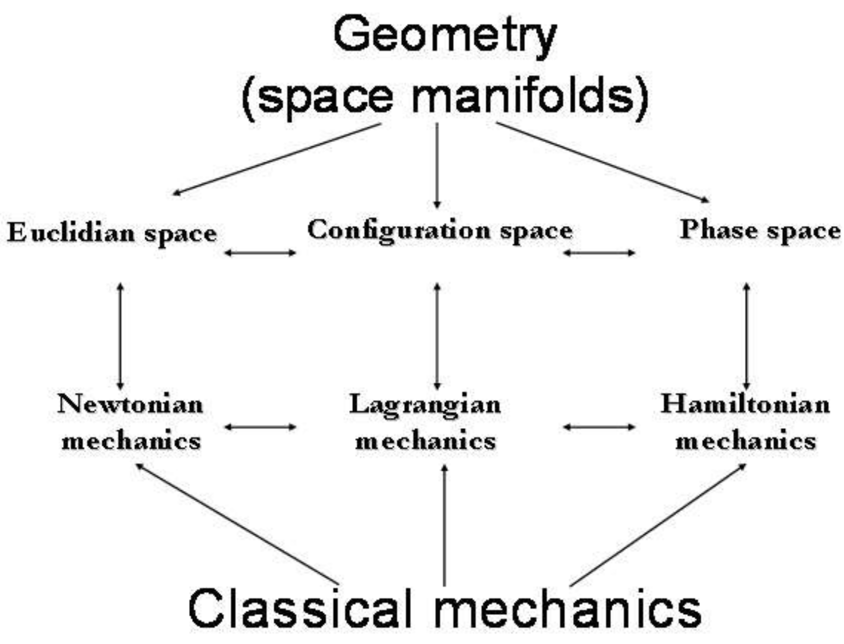

Figure 1 – The scheme of mutual transitions among different spaces and respective mechanics

The approach used will undoubtedly contribute to the formation of a holistic picture of the world among students. In this case, the “fragmentation” of the studied concepts disappears, and all of them, united by geometric images, merge into something single [5]. In the classroom, it becomes much more interesting to discuss and find out in which space it is better to consider the object, and which parameters will most fully describe its movement and state over time. In addition, this approach will allow students to better understand all subsequent sections of theoretical physics.

In conclusion, we can note the fact that geometric principles that were actively used earlier in mathematical physics [6], various theories of gravity and cosmology (see, for example, [7]) and field theory [8-9], have now paved the way even in thermodynamics [10-11]. Thus, the introduction of geometric principles into the physics course is fundamentally important for the preparation of highly qualified specialists in the field of both theoretical and applied physics.

References

1 Aryngazin, K. M. Geometrical ideas in theoretical physics [Text] / K. M. Aryngazin // Almaty, 1994. – 360 p.

2 Dubrovin, B. A. Modern geometry. Methods and application. / B. A. Dubrovin, S. P. Novikov, A. T. Fomenko // Moscow, 1979. – 760 p.

3 Pereira, J. G. An Introduction to Geometrical Physics [Text] / J. G. Pereira, R. Aldrovandi // World Scientific Publishing Company, 2017. – 812 p.

4 Yefremov, A. P. Physical theories in hypercomplex geometric description [Text] / A. P. Yefremov // International Journal of Geometric Methods in Modern Physics. – 2014. – Vol. 11 – № 6. – P. 33.

5 Aryngazin, K. M. Methodological problems of the theoretical physics formation / K. M. Aryngazin, I. F. Vassilyeva // Bulletin of Karaganda university. Series “Physics”. – 2010. – № 3(59). – P. 56-61.

6 Schutz, B. F. Geometrical Methods of Mathematical Physics [Text] / Bernard F. Schutz // Cambridge University Press, 1980. – 250 p.

7 Arkhipov, V. V. Geodesics in a Two-Dimensional Model of Closed Spacetime [Text] / V. V. Arkhipov // Russian Physics Journal. – 2014. – Vol. 57. – № 8. – P. 1023-1029.

8 Sokolik, H. Geometrical description of fields [Text] / H. Sokolik // Il Nuovo Cimento A. – 1979. – Vol. 51. – Iss. 3. – P. 339-344.

9 Arkhipov, V. V. Minimal Cohomological Model of a Scalar Field on a

Riemannian Manifold [Text] / V. V. Arkhipov // Russian Physics Journal. – 2018. – Vol. 60. – Iss. 12. – P. 2051-2062.

10 Hawking, S. W. Thermodynamics of black holes in anti-de Sitter space [Text] / S. W. Hawking, D. N. Page // Communications in Mathematical Physics. – 1983. – Vol. 87. – № 4. – P. 577–588.

11 Larrañaga, A. Geometric Description of the Thermodynamics of the Noncommutative Schwarzschild Black Hole [Electronic source] / A. Larrañaga, N. Herrera, J. Garcia // Advances in High Energy Physics. – 2013. – 6 p. Article ID 641273.